Hyperfine Interact (2019) 240:58
https://doi.org/10.1007/s10751-019-1603-x

Quantum reflection of antihydrogen from a liquid helium bulk

P.-P. Crépin[1] · E. A. Kupriyanova[2,3] · R. Guérout[1] · A. Lambrecht[1] · V. V. Nesvizhevsky[4] · S. Reynaud[1] · S. Vasiliev[5] · A. Yu. Voronin[2,3]

Published online: 20 May 2019

Abstract
We study the quantum reflection of ultracold antihydrogen atoms bouncing on the surface of a liquid helium bulk. The Casimir-Polder potential and quantum reflection are calculated and compared to the same quantities for other bulks. Antihydrogen can be protected from annihilation for as long as 1.3 s on a bulk of liquid ^{4}He, and 1.7 s for liquid ^{3}He. These large lifetimes open interesting perspectives for spectroscopic measurements of the free fall acceleration of antihydrogen.

Keywords Quantum reflection · Antihydrogen · Liquid helium

1 Introduction

Quantum reflection is a non classical phenomenon which appears when a quantum matter wave approaches a rapidly varying attractive potential. Instead of accelerating towards the surface, the quantum particle has a probability to be reflected. This process has been studied

This article is part of the Topical Collection on *Proceedings of the 13th International Conference on Low Energy Antiproton Physics (LEAP 2018) Paris, France, 12–16 March 2018*
Edited by Paul Indelicato, Dirk van der Werf and Yves Sacquin

✉ P.-P. Crépin
pierre-philippe.crepin@lkb.upmc.fr

R. Guérout
romain.guerout@lkb.upmc.fr

S. Reynaud
serge.reynaud@lkb.upmc.fr

1 Laboratoire Kastler Brossel (LKB), Sorbonne Université, CNRS, ENS-PSL Université, Collège de France, 75252, Paris, France

2 P.N. Lebedev Physical Institute, 53 Leninsky Prospect, 117924, Moscow, Russia

3 Russian Quantum Center, 100 A, Novaya Street, Skolkovo, 143025, Moscow, Russia

4 Institut Laue-Langevin (ILL), 71 Avenue des Martyrs, 38042, Grenoble, France

5 Department of Physics and Astronomy, University of Turku, 20014, Turun Yliopisto, Finland

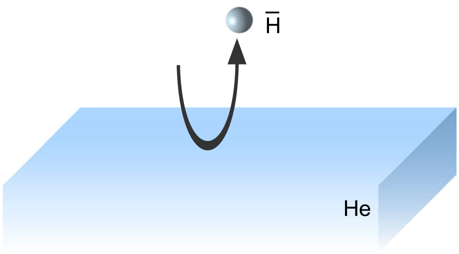

Fig. 1 Representation of the quantum reflection process for an antihydrogen atom falling onto a helium bulk

theoretically for the van der Waals potential since the early days of quantum mechanics [1–4]. It was first observed experimentally for H and He atoms [5–7] and then for ultracold atoms or molecules on solid surfaces [8–11].

In the last years quantum reflection has been studied also for antimatter [12–14] since it should play a key role in experiments with antihydrogen atoms [15–17]. It was shown that the free fall acceleration of antihydrogen can in principle be evaluated accurately [18] through spectroscopic studies of the quantum levitational states [19, 20] of atoms trapped by quantum reflection and gravity [21, 22]. Following uncertainty principle of quantum mechanics, such spectroscopic measurements should have a better accuracy for larger lifetime of antihydrogen in the trap.

We calculate quantum reflection of antihydrogen above a liquid helium bulk and show that the lifetime of antihydrogen reaches values as high as 1.3 s for a bulk of liquid ^{4}He and 1.7 s for a bulk of liquid ^{3}He.

2 Casimir-Polder potential

We study quantum reflection for an antihydrogen atom of mass m falling onto a liquid helium bulk (see Fig. 1).

The atom is sensitive to the Casimir-Polder (CP) potential and to the free fall acceleration $\overline{g}$. We focus our attention on the quantum reflection in the CP potential,[1] as the latter is effective at distances z much smaller than the length scale ℓ_g associated with quantum effects in the gravity field $\ell_g = (\hbar^2/2m^2 g)^{1/3} \simeq 5.87\,\mu$m. Note that we suppose $\overline{g} = g$ in all numerical evaluations.

The CP potential is evaluated at zero temperature as the following function of the distance z of the atom to the liquid helium surface [23]

$$V(z) = \frac{\hbar}{c^2}\int_0^\infty \frac{d\xi}{2\pi}\xi^2\alpha(i\xi)\int \frac{d^2\mathbf{q}}{(2\pi)^2}\frac{e^{-2\kappa z}}{2\kappa}\sum_{p=TE,TM} s_p r_p,$$

$$\kappa = \sqrt{\xi^2/c^2 + \mathbf{q}^2}\,, \quad s_{\mathrm{TE}} = 1\,, \quad s_{\mathrm{TM}} = -\frac{\xi^2 + 2c^2\mathbf{q}^2}{\xi^2}. \tag{1}$$

The dynamic polarizability $\alpha(\omega)$ of the antihydrogen atom is supposed to be the same as for the hydrogen atom. The reflection amplitudes r_p are calculated for polarizations p by combining the Fresnel amplitudes at interfaces and propagation in the helium bulk. The optical properties of ^{4}He are described with a sufficient accuracy by a model dielectric constant with three resonances [24, 25].

[1] The full problem of combined effects of the CP and gravity potentials is treated in [18].

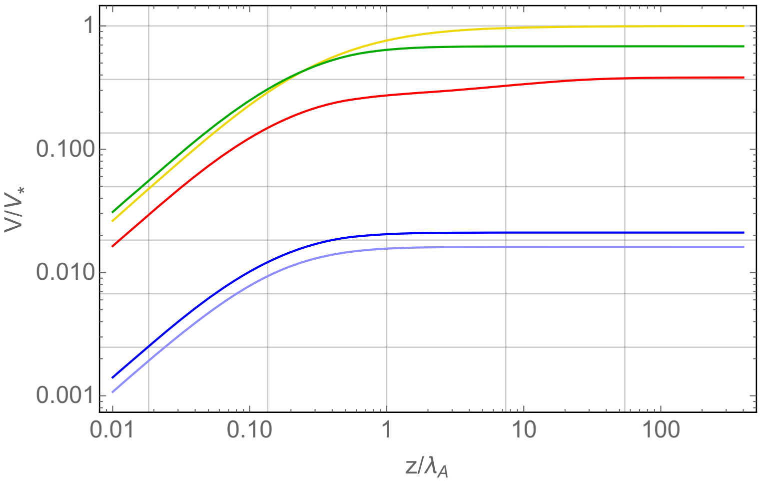

Fig. 2 CP potentials $V(z)$ normalized by the potential $V_*(z) \simeq -C_4/z^4$ calculated for a perfect mirror at large distances. Distances z are normalized by the wavelength $\lambda_A \simeq 121$nm of the 1S→2P antihydrogen transition. The full lines correspond, from bottom to top, to bulks of ^{3}He (light blue), ^{4}He (dark blue), silica (red), silicon (green) and gold (yellow)

It follows that quantum reflection occurs closer to the material surface where the CP potential is much steeper, which explains the large quantum reflection probability found below, with reflection even larger for ^{3}He than for ^{4}He [25]. In both cases we use an effective dielectric constant and disregard the role played by excitations in the helium like ripplons. The latter is well justified at temperatures below 100 mK [26], the temperature range where results obtained in the following are accurate.

3 Quantum reflection

The calculations show a very low value of the CP potential (Fig. 2), as explained by the fact that liquid helium is almost transparent for the electromagnetic field. We now discuss the consequence of this fact in terms of large quantum reflection from liquid helium bulks.

To this aim, we solve the Schrödinger equation for the antihydrogen falling into the CP potential [15] above the liquid helium bulk. We then obtain the reflection amplitude r as the ratio of the outgoing wave to the incoming one far from the bulk. The quantum reflection probability is the squared modulus of this amplitude $R = |r|^2$. The results are shown in Fig. 3 with larger and larger probability obtained for the weaker and weaker potentials of Fig. 1. In particular, quantum reflection for atoms falling from a height h and thus having a given energy $E = mgh$ is much larger on a liquid helium bulk than on the other materials studied here.

We extract from the reflection amplitude the *scattering length* $a \equiv -\frac{i}{k}\frac{1+r(k)}{1-r(k)}$, $k \to 0$. For quantum reflection on a liquid ^{4}He bulk, one finds for example $a = -(34.983 + 44.837i)\, a_0$. The imaginary part $b = -\mathrm{Im}(a)$ of this scattering length determines the mean lifetime τ for atoms bouncing above the bulk [18]: $\tau = \frac{\hbar}{2mgb}$. In Table 1, we compare this values obtained for τ from the quantum reflection propabilities drawn on Fig. 3 and also for porous silica studied in [16].

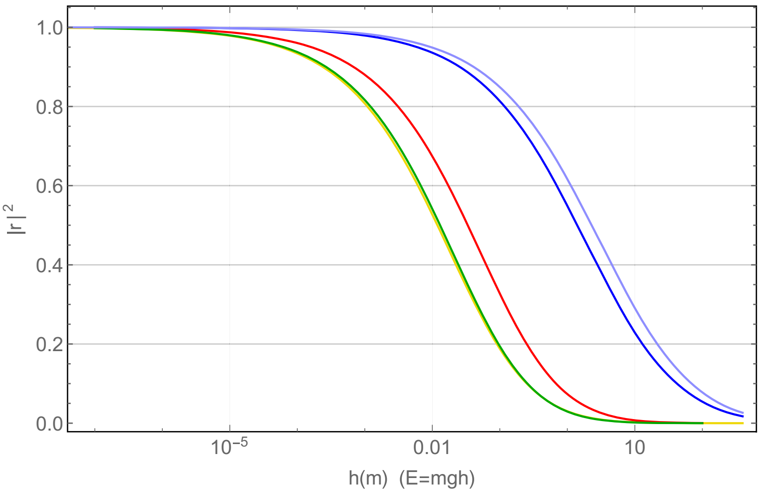

Fig. 3 Quantum reflection probability as a function of the free fall height of the atom h, that is also of its energy $E = mgh$. The full lines correspond, from top to bottom, to bulks of ^{3}He (light blue), ^{4}He (dark blue), silica (red), silicon (green) and gold (yellow)

Table 1 Lifetime τ of antihydrogen in seconds above various material surfaces and number N_1 of semi-classical bounces for an atom in the first quantum gravitational state for different bulk materials and for porous silica (see [16] for the latter case)

Material	τ [s]	N_1
perfectly reflective	0.11	33
silica bulk	0.22	66
porous silica (95% porosity)	0.94	282
liquid ^4He bulk	1.35	405
liquid ^3He bulk	1.71	514

We also give the values for the number N_1 of semi-classical bounces for an atom in the first quantum levitation state. The numbers show that liquid helium is a much better reflector for antihydrogen matter waves than the other materials which have been studied up to now. The much larger lifetime, that is also the much larger number of bounces before annihilation, implies that it should be possible to trap antimatter for long enough to improve significantly the spectroscopy measurements discussed in [18].

4 Conclusion

In this letter we found theoretically a high reflection probability for antihydrogen atoms falling down onto liquid helium bulk.

The long lifetime obtained for antimatter above a liquid helium bulk can have very interesting applications for new spectroscopic tests of the equivalence principle for antihydrogen atoms.

In a free fall time experiment, we can make use of the possibility of bounces to select first gravitational states and then reduce the uncertainty by shaping vertical velocity distribution [17, 27, 28]. Another possibility is to perform spectroscopic measurements of these quantum gravitation states for example by using an oscillating magnetic field [28], from which the gravitational acceleration can be extracted [18].

The influence of the finite thickness of a liquid helium film to the quantum reflection and to the variant scattering length is studied in [25].

Acknowledgements We are grateful to our colleagues from GBAR and GRANIT collaborations for useful discussions.

References

1. Lennard-Jones, et al.: Proc. R. Soc. London, Ser. A **156**, 6–28 (1936)
2. Lennard-Jones, et al.: Proc. R. Soc. London, Ser. A **156**, 29–36 (1936)
3. Berry, M.V., Mount, K.E.: Rep. Progr. Phys. **35**, 315–397 (1972)
4. Friedrich, H., Trost, J.: Phys. Rep. **297**, 359–449 (2004)
5. Nayak, V.U., et al.: Phys. Rev. Lett. **50**, 990–992 (1983)
6. Berkhout, J.J., et al.: Phys. Rev. Lett. **63**, 1689–1692 (1989)
7. Yu, I.A., et al.: Phys. Rev. Lett. **71**, 1589–1592 (1993)
8. Shimizu, F.: Phys. Rev. Lett. **86**, 987–990 (2001)
9. Druzhinina, V., DeKieviet, M.: Phys. Rev. Lett. **91**, 193202 (2003)
10. Pasquini, T.A., et al.: Phys. Rev. Lett. **93**, 223201 (2004)
11. Pasquini, T.A., et al.: Phys. Rev. Lett. **97**, 093201 (2006)
12. Voronin, A.Y., et al.: Phys. Rev. A **72**, 062903 (2005)
13. Voronin, A.Yu., et al.: Phys. Rev. A **83**, 032903 (2011)
14. Voronin, A.Yu., et al.: Phys. Rev. A **85**, 014902 (2012)
15. Dufour, G., et al.: Phys. Rev. A **87**, 012901 (2013)
16. Dufour, G., et al.: Phys. Rev. A **87**, 022506 (2013)
17. Dufour, G., et al.: Eur. Phys. J. C **74**, 2731 (2014)
18. Crépin, P.-P., et al.: Phys. Rev. A **95**, 032501 (2017)
19. Breit, G.: Phys. Rev. **32**, 273–276 (1928)
20. Nesvizhevsky, V.V., et al.: Nature **415**, 297–299 (2002)
21. Jurisch, A., Friedrich, H.: Phys. Lett. A **349**, 230–235 (2006)
22. Madroñero, J., Friedrich, H.: Phys. Rev. A **75**, 022902 (2007)
23. Messina, R., et al.: Phys. Rev. A **80**, 022119 (2009)
24. Sabisky, E.S., Anderson, C.H.: Phys. Rev. A **7**, 790 (1972)
25. Crépin, P.-P., et al.: Europhys. Lett. **119**, 33001 (2017)
26. Goldman, V.V.: Phys. Rev. Lett. **56**, 612–615 (1986)
27. Voronin, A.Yu., et al.: J. Phys. B: At. Mol. Opt. Phys. **49**, 054001 (2016)
28. Dufour, G., et al.: Adv. High Energy Phys. **2015**, 379642 (2015)